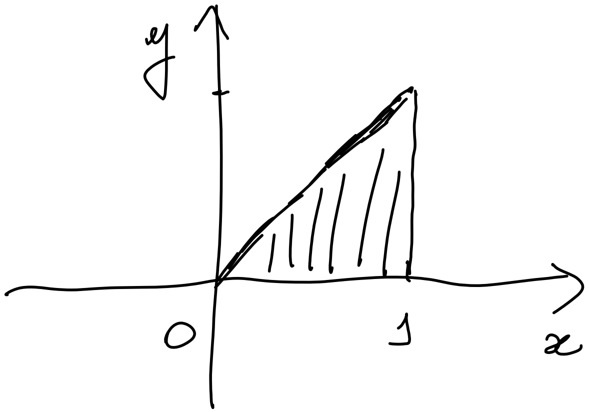
Corrigé Probas 2012 + Rattrapage

III

Mai 2012

$$f(x,y) = \begin{cases} Cxy & (0 \leq y \leq x \leq 1) \\ 0 & \text{ailleurs} \end{cases}$$

Support :



a) $$\iint_{\mathbb{R}^2} f(x,y)\,dx\,dy = 1 = \int_0^1 dx \int_0^x Cxy\,dy = \int_0^1 Cx\,dx \left[\frac{1}{2}y^2\right]_0^x$$

$$\Rightarrow \int_0^1 \frac{C}{2} x^3\,dx = 1 = C\left[\frac{x^4}{8}\right]_0^1 = \frac{C}{8} \Rightarrow \underline{C = 8}$$

b) $$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 0 & \text{si } x \notin [0,1] \\ \displaystyle\int_0^x 8xy\,dy = 4x\left[y^2\right]_0^x = 4x^3 & \text{si } x \in [0,1] \end{cases}$$

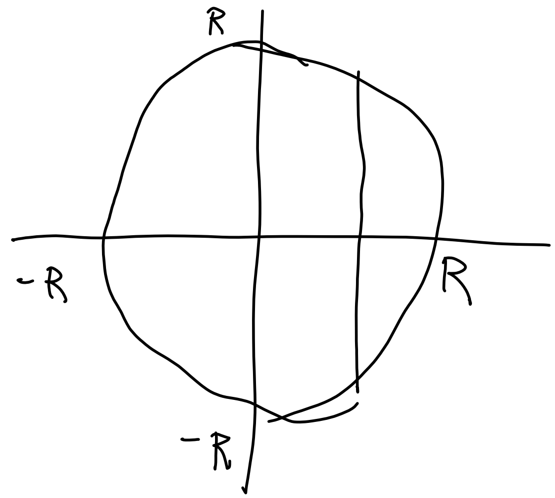
$$f_X(x) = \begin{cases} 4x^3 & (0 \leq x \leq 1) \\ 0 & \text{ailleurs} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} 0 & \text{si } y \notin [0,1] \\ \displaystyle\int_y^1 8xy\,dx = 4y\left[x^2\right]_y^1 = 4y(1-y^2) & \text{si } 0 \leq y \leq 1 \end{cases}$$

$$f_Y(y) = \begin{cases} 4y(1-y^2) & (0 \leq y \leq 1) \\ 0 & \text{ailleurs} \end{cases}$$

c) $f(x,y) \neq f_X(x)\, f_Y(y) \Rightarrow$ <u>X et Y ne sont pas indépendantes</u>

II Juin 2012



a) $$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = 0 \quad \text{si } |x| > R$$

Si $|x| < R$ $$f_X(x) = \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2}\,dy$$

$$f_X(x) = \begin{cases} \dfrac{2}{\pi R^2} \sqrt{R^2 - x^2} & \text{si } |x| \leq R \\ 0 & \text{ailleurs} \end{cases}$$

De même

$$f_Y(y) = \begin{cases} \dfrac{2}{\pi R^2} \sqrt{R^2 - y^2} & \text{si } |y| \leq R \\ 0 & \text{ailleurs} \end{cases}$$

$$f_{XY}(x,y) \neq f_X(x)\, f_Y(y) \Rightarrow$$ X et Y ne sont pas indépendantes

b) $$E(X) = \int_{-R}^{R} \underbrace{\frac{2x}{\pi R^2} \sqrt{R^2 - x^2}}_{g(x) = -g(-x)}\, dx = \int_{-R}^{0} g(x)\, dx + \int_{0}^{R} g(x)\, dx$$

$x \to -x$

$$= \int_0^R -g(x)\,dx + \int_0^R g(x)\,dx = 0$$

Idem pour $Y \Rightarrow E(X) = E(Y) = 0$

c) $\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = E(XY)$

$$= \iint_D \frac{xy}{\pi R^2}\,dx\,dy$$

Coordonnées polaires $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ $\quad 0 \leq r \leq R$, $\quad 0 \leq \theta < 2\pi$

$$dx\,dy = r\,dr\,d\theta$$

$$\mathrm{Cov}(X,Y) = \int_0^{2\pi} d\theta \int_0^{R} r^3 \sin\theta\cos\theta \, dr$$

$$= \frac{1}{2}\int_0^{2\pi} \sin 2\theta \, d\theta \int_0^{R} r^3 \, dr = \underbrace{\left[-\frac{1}{4}\cos 2\theta\right]_0^{2\pi}}_{=0} \frac{R^4}{4} = 0$$

$$\Rightarrow \underline{\mathrm{Cov}(X,Y) = 0}$$

d) $\mathrm{Cov}(X,Y) = 0 \not\Rightarrow$ X et Y sont indépendantes