

Optimisation linéaire - TD1

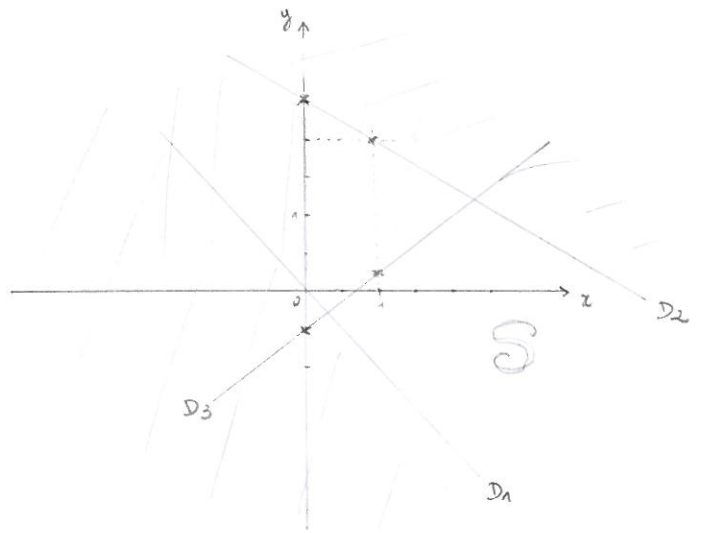
Exercice 1:

$$a = \begin{cases} 2x + 4y \leq 10 \\ 3x - 4y \geq 2 \\ x + y \geq 0 \end{cases}$$

$$D_1: y = -x$$

$$D_2: y = \frac{10 - 2x}{4} = -\frac{1}{2}x + \frac{5}{2}$$

$$D_3: y = \frac{2 - 3x}{-4} = -\frac{2}{4} + \frac{3}{4}x$$

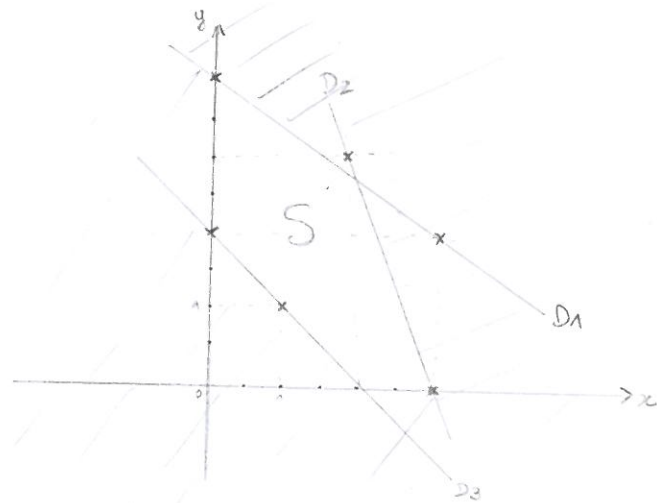


$$b = \begin{cases} 2x + 3y \leq 12 \\ 3x + y \leq 9 \\ x + y \geq 2 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$$D_1: y = \frac{12 - 2x}{3} = 4 - \frac{2}{3}x$$

$$D_2: y = 9 - 3x$$

$$D_3: y = 2 - x$$



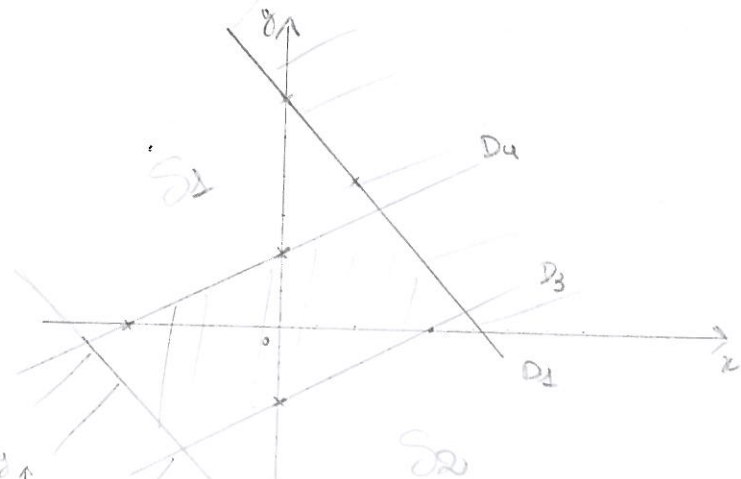
$$c = \begin{cases} |x + 2y| \leq 6 \\ |x - 2y| \geq 2 \end{cases}$$

$$D_{11}: x + 2y = 6 \Rightarrow y = \frac{6 - x}{2} = 3 - \frac{x}{2}$$

$$D_{12}: -x - 2y = 6 \Rightarrow y = \frac{-6 - x}{2} = -3 - \frac{x}{2}$$

$$D_{21}: x - 2y = 2 \Rightarrow y = \frac{-2 + x}{2} = -1 + \frac{x}{2}$$

$$D_{22}: -x + 2y = 2 \Rightarrow y = \frac{2 + x}{2} = 1 + \frac{x}{2}$$



Exercice 2: max(Z = x + 3y)

$$\text{avec: } \begin{cases} 2x + 5y \leq 10 \\ 3x + 4y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$$\hookrightarrow D_0: y = -\frac{x}{3}$$

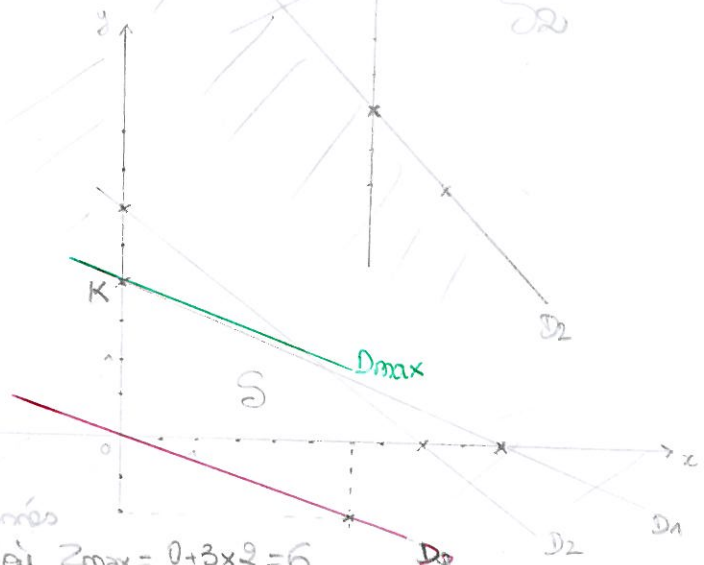
$$D_1: y = \frac{10 - 2x}{5} = 2 - \frac{2}{5}x$$

$$D_2: y = \frac{12 - 3x}{4} = 3 - \frac{3}{4}x$$

$$\text{pour } x=0: \\ y = 2 \cdot \frac{2 \cdot 0}{5} = 2$$

K(0;2) point d'intersection entre la droite des données et D1

$$\text{D'où } Z_{\max} = 0 + 3 \times 2 = 6$$



Exercice 3:

① $\min(z = 2x + 3y)$

avec:

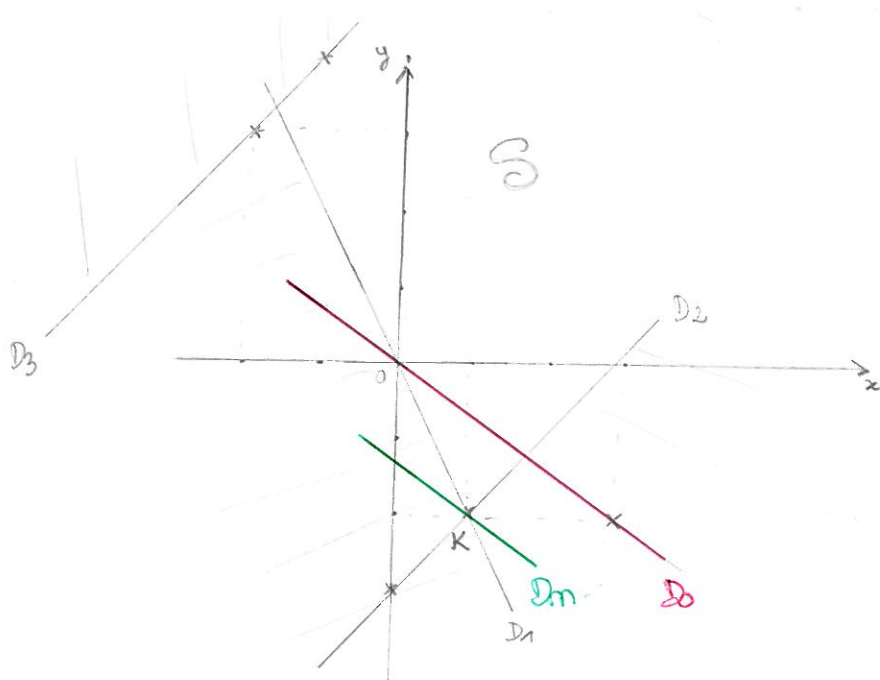
$$\begin{cases} 2x + y \geq 0 \\ x - y \leq 3 \\ x - y \geq -5 \end{cases}$$

$D_1: y = -2x$

$D_2: y = -3 + x$

$D_3: y = 5 + x$

$D_0: 2x + 3y = 0 \Rightarrow y = -\frac{2x}{3}$



K intersection de D_1 et D_2 :

$$\begin{cases} 2x + y = 0 \\ x - y = 3 \end{cases} \Rightarrow \begin{cases} 2(3+y) + y = 0 \\ x = 3+y \end{cases} \Rightarrow \begin{cases} 6 + 2y + y = 0 \Rightarrow y = -\frac{6}{3} = -2 \\ x = 3 - 2 = 1 \end{cases}$$

D'où $Z_{\min} = 2 \times 1 + 3 \times (-2) = 2 - 6 = -4$

② $\max(z = 2x + 3y)$

↳ infini, on ne peut le déterminer.

Exercice 5:

$\max(z = 10x_1 + 30x_2)$

avec

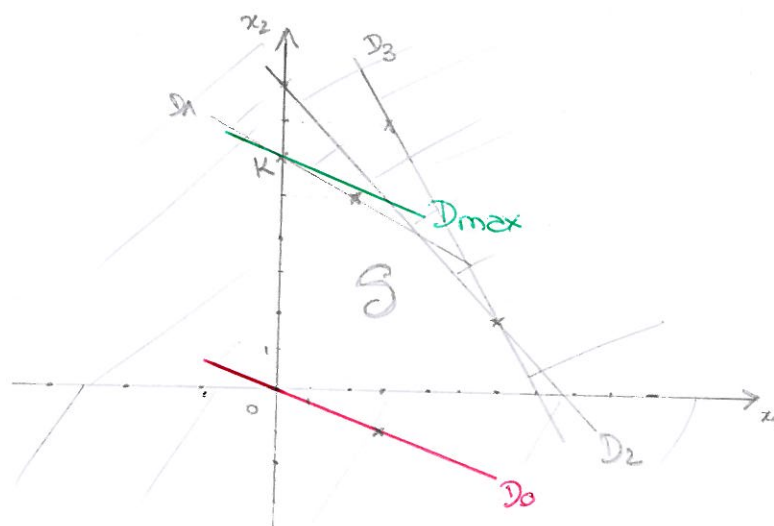
$$\begin{cases} x_1 + 3x_2 \leq 18 \\ x_1 + x_2 \leq 8 \\ 2x_1 + x_2 \leq 14 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

$D_1: x_2 = \frac{18 - x_1}{3} = 6 - \frac{x_1}{3}$

$D_2: x_2 = 8 - x_1$

$D_3: x_2 = 14 - 2x_1$

$D_0: x_2 = -\frac{10x_1}{30} = -\frac{1}{3}x_1$



K: intersection entre $(y=0)$ et D_1 :

$x_2 = 6 \Rightarrow K(0; 6)$

D'où $Z_{\max} = 30 \times 6 = 180$

Exercice 4:

$\max(z = 40x + 50y)$

Avec:

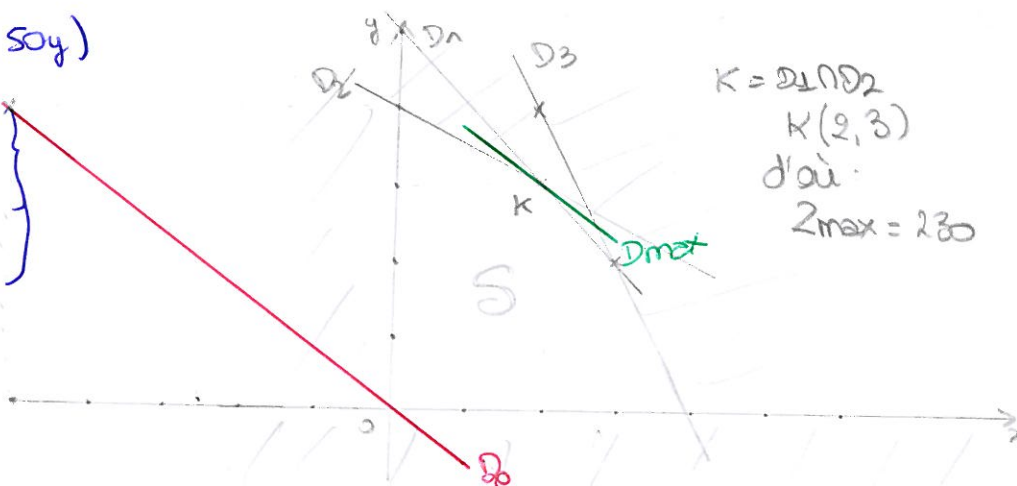
$$\begin{cases} 10x + 10y \leq 50 \\ 10x + 20y \leq 80 \\ 20x + 10y \leq 80 \\ x \geq 0, y \geq 0 \end{cases}$$

$D_1: y = \frac{50 - 10x}{10} = 5 - x$

$D_2: y = \frac{80 - 10x}{20} = 4 - \frac{1}{2}x$

$D_3: y = \frac{80 - 20x}{10} = 8 - 2x$

$D_0: y = -\frac{40}{50}x = -\frac{4}{5}x$



$K = D_1 \cap D_2$

$K(2; 3)$

d'où:

$Z_{\max} = 230$