Ing1 2007-2008
Remise à niveau - ANALYSE

Série 1

Révisions : Intégrales

Exercice 1
Calculer les primitives suivantes :

$$\begin{aligned} P_1 &= \int \frac{dx}{x^2+5} \quad P_2 = \int \frac{dx}{x^2-5} \quad P_3 = \int e^x \sin(e^x) dx \quad P_4 = \int \tan^3 x dx \\ P_5 &= \int \frac{dx}{\tan^3 x} \quad P_6 = \int \frac{2x+3}{(x^2+3x+7)^m} dx, m \in \mathbb{N} \quad P_7 = \int \frac{\ln x}{x} dx \quad P_8 = \int \frac{\operatorname{ch} x}{\operatorname{sh}^5 x} dx \end{aligned}$$

Exercice 2
Considérons l'intégrale

$$I = \int_0^{\ln 2} \sqrt{e^x - 1} dx$$

Effectuer le changement de variable $u = \sqrt{e^x - 1}$ et calculer I (réponse $I = 2 - \pi/2$)

Exercice 3
Calculer les primitives suivantes :

$$\begin{aligned} P_1 &= \int \frac{1}{\sqrt{2+x} + \sqrt[3]{2+x}} dx, (t = \sqrt[6]{2+x}) \\ P_2 &= \int \frac{1}{((x-1)^2-4)^2} dx, (\operatorname{th} u = \frac{x-1}{2} \text{ ou } \coth u) \\ P_3 &= \int (\arcsin x)^2 dx \qquad P_4 = \int x^2 \sqrt{1+x^3} dx \end{aligned}$$

Exercice 4
Calculer les primitives suivantes :

$$\begin{aligned} Q_1 &= \int (\cos x \cos 2x + \sin x \sin 3x)\, dx \qquad Q_2 = \int \cos x \sin^4 x dx \\ Q_3 &= \int \cos^6 x dx \qquad Q_4 = \int \sin^3 x \cos^2 x dx \qquad Q_5 = \int \operatorname{ch}^2 x \operatorname{sh}^2 x dx \end{aligned}$$

Exercice 5
Décomposer si nécessaire les fractions rationnelles, puis en calculer les primitives :

$$\begin{aligned} R_1 &= \int \frac{4x}{(x-2)^2} dx \qquad R_2 = \int \frac{1}{x^2+x+1} dx \\ R_3 &= \int \frac{1}{x^3+1} dx \qquad R_4 = \int \frac{x+1}{x\,(x-2)^2} dx \end{aligned}$$

Solutions : $R_1 = -\frac{8}{x-2} + 4\ln(x-2) + \lambda$; $R_2 = \frac{2}{3}\sqrt{3} \arctan \frac{1}{3}(2x+1)\sqrt{3} + \lambda$; $R_3 = \frac{1}{3}\ln(1+x) - \frac{1}{6}\ln\left(x^2 - x + 1\right) + \frac{1}{3}\sqrt{3}\arctan\frac{1}{3}(2x-1)\sqrt{3} + \lambda$; $R_4 = \frac{1}{4}\ln x - \frac{3}{2(x-2)} - \frac{1}{4}\ln(x-2) + \lambda$

Exercice 6

Calculer les intégrales suivantes :

$$\begin{aligned} I_1 &= \int_0^1 \frac{dx}{x^2+2} \qquad I_2 = \int_{-1/2}^{1/2} \frac{dx}{1-x^2} \qquad I_3 = \int_2^3 \frac{2x+1}{x^2+x-3}dx \\ I_4 &= \int_0^2 \frac{x}{x^4+16}dx \qquad I_5 = \int_{-2}^{0} \frac{1}{x^3-7x+6}dx \end{aligned}$$

Solutions : $I_1 = -\frac{1}{4}i\sqrt{2}\ln\left(2+i\sqrt{2}\right) + \frac{1}{4}i\sqrt{2}\ln\left(2-i\sqrt{2}\right) = \frac{1}{\sqrt{2}}\arctan\frac{1}{\sqrt{2}}$
$I_2 = 2\,\mathrm{arctanh}\,\frac{1}{2} = \ln 3$; $\quad I_3 = \ln 3$; $\quad I_4 = \frac{1}{32}\pi$; $\quad I_5 = -\frac{1}{5}\ln 2 + \frac{3}{10}\ln 3 = \frac{1}{10}\ln(27/4)$