

Metaheuristics for the 3D Bin Packing Problem in the Steel Industry

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Abstract—This work presents heuristic and metaheuristic approaches for addressing the real-world steel cutting problem of a retail steel distributor as a cutting & packing problem. It consists of the cutting of large steel blocks in order to obtain smaller pieces ordered by clients. The problem was formulated as a 3-dimensional residual bin packing problem for minimization of scrap generation, with guillotine cutting constraint and chips scrap generation. A tabu search and best-fit decreasing (BFD) approaches are proposed and their performance compared to an heuristic and ant colony optimization (ACO) algorithms. It's shown that the tabu search and best-fit decreasing algorithm are able to reduce the generated scrap by up to 52% in comparison with the heuristic in [1]. The orders to suppliers were also reduced by up to 35%. The analysis of the results of the different approaches provide insight onto the most important factors in the problem's scrap minimization.

I. INTRODUCTION

The Cutting & Packing (C&P) class of problems is present in a wide number of industry and services applications such as the cutting of paper, metal, glass, basic pallet loading and container loading. Early work in C&P problems has been referred to have started before the 20th century [2]. A C&P problem is structured by a set of large objects (input, supply) and a set of small items (output, demand), which are defined in n geometric dimensions. The objective of this type of problem is to assign the small items to the large objects to optimize a given objective function, such as certain constraints are satisfied.

This work focuses on the problem of assigning the retail demand of steel blocks (set of boxes) to a set of inventory steel blocks (set of bins), from which the former should be cut, where both sets are strongly heterogeneous. The objective is to minimize the scrap metal. According to the most recent C&P typology [3], this problem is considered a 3D residual bin packing problem with the specificity of the use of a timeline. There is a variable daily set of boxes to be cut from a set of bins that suffers changes resulting from the cuts.

In Miguel Silveira et al. [1] this problem was approached using different scrap conditions and without considering chip

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generation. Good results were achieved using a first-fit decreasing (FFD) type heuristic and a ant colony optimization (ACO) approach.

C&P problems have been addressed using a variety of approaches. Heuristic approaches like first-fit and best-fit decreasing are popular due to their simplicity and ability to achieve good results. These have also been used as the basis for more complex approaches based on metaheuristics. [4, 5, 6]

Tabu-search [4, 6, 7], ACO [8], genetic [9] and branch-and-bound [10] algorithms have also been used with success. Mixed integer programming using column-based approaches achieve very good results [5] but are more suited for problems with smaller sets of boxes and bins that minimize the number of possible cutting patterns.

In the followed typology of C&P problems [3], dated from 2007, only two studies are found for the residual bin packing problem (RBPP). More recently, a 3D residual bin packing problem was studied by Bang-Jensen [11], proving the increasing need of more complex algorithms, i.e. for three-dimensional problems with multiple bins, in order to solve real-world problems.

This work proposes a tabu search and a best-fit decreasing type algorithms for the 3D residual bin packing problem with guillotine constraint. These approaches search for the best bin and packing point to place each box and use an heuristic algorithm (3DHA) as the basis to construct solutions. The tabu search approach searches for benefits on the combined analysis of the packing of multiple boxes. The algorithms are applied to a real-world problem in the steel industry. Their performance is compared to the 3DHA and ACO algorithm presented in [1].

This paper is organized as follows. Section II presents the formulation of the addressed problem and its classification according to the most recent typology, section III briefly explains the heuristic used for solution construction, section IV explains and references the ACO algorithm used and, in section V the proposed tabu search and best-fit decreasing algorithms are presented. In section VI the application example data and results are presented and section VII concludes this paper and points to future work.

II. STEEL CUTTING PROBLEM

Arising from a steel retailer company, the addressed problem consists of the cutting of large steel blocks coming from steel plants, in order to obtain the smaller tailored blocks that are made on demand to supply different industries like

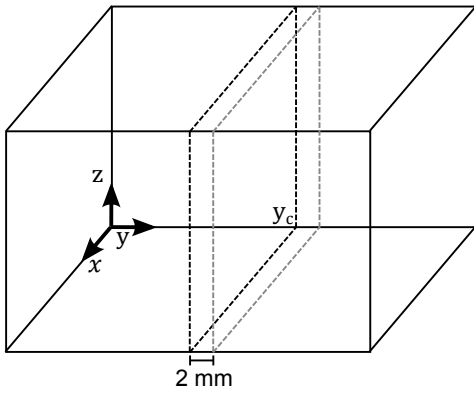


Fig. 1. Chip representation during a cutting operation at y_c position.

the tooling industry. The optimization of the order to stock piece assignment and cutting can have significant economical effects, not only because the cutting should be minimal, but also due to the residual bins resulting from the cuts with dimensions that make them be characterized as scrap.

This problem consists of executing orthogonal cuts for a set of rectangular-shaped boxes from rectangular-shaped bins. Each box j ($j = 1, \dots, n$) is characterized by length l , width w and depth d , where $l \geq w \geq d$. The bins are characterized by length L , width W and depth D , where $L \geq W \geq D$. All orthogonal orientations of the boxes are allowed. The coordinates origin is located at the bottom-left-behind of the bin and (x_j, y_j, z_j) represents the point where the bottom-left-behind point of the box j is placed.

In order to have a feasible arrangement, the following 3-dimensional packing constraints have to be met:

- 1) Each box is placed completely within the bin
- 2) A box i may not overlap another box j
- 3) Each box is placed parallel to the side walls of the bins (orthogonal packing)

The clients orders, which represent the boxes, can have a associated quantity greater than one. The steel cutting process is characterized by the guillotine cut constraint, which means that all the cuts have to be edge-to-edge.

A. Chips Generation

When processing a saw steel cutting operation, part of the material is lost in the form of chips. The chips have approximately the dimension of the cut area with a 2 millimeter thickness. When a cut is made, a volume of $A_c \times 2$ (mm^3) is lost (A_c being the cut area). Figure 1 exemplifies the chip volume during a cutting operation between the areas limited by dashed lines.

After a cutting operation is finished, the leftover pieces (residual bins) that are not considered useful for future cuts and do not compensate to be kept in stock are sold back to the steel plants for recycling. A residual bin, this is, a volume of material obtained after the cutting of a bin, is considered scrap metal, and not kept in stock (P_{stk}), if it complies to any of the conditions presented in equations 1, 2 and 3.

B. Scrap Metal Definition

$$L \leq 375, 150 \leq W \leq 300, 151 \leq D \leq 300 \quad (mm) \quad (1)$$

$$L \leq 1950, W \leq 225, D \leq 120 \quad (mm) \quad (2)$$

$$weight \leq 35 \quad (kg) \quad (3)$$

The objective of the optimization is to minimize the quantity of scrap generated in the cutting process, this means minimizing the generation of residual bins that comply to any of the above scrap conditions and minimizing the cutting area. The objective function is equal to the sum of the scrap residual bins and chips generated from the cutting operations.

The considered problem is divided into several small daily problems according to the daily demand, in which the final stock of one day becomes the initial stock of the next day.

The cutting decision system presented in figure 2 represents the problem. With the orders and stock as input and the cuts to execute in order to minimize the objective as the desired output. Its function is to make the piece association for client's orders in order to minimize the scrap metal generated. The proposed decision system is developed in order to simultaneously decide on the cutting of various orders. The analysis of multiple orders instead of one at a time was made to conclude on the benefits of joint order optimization.

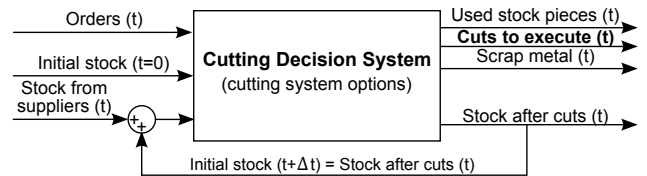


Fig. 2. Cutting decision system

C. Problem Classification

The following presents the steel cutting problem characteristics classification according to Wäscher's typology [3].

- Dimensionality - Three dimensions are used to characterize the problem.
- Kind of assignment - As all orders have to be fulfilled in order to minimize a function related to the bins used; the problem is considered of the "input minimization" type.
- Assortment of boxes - The assortment of all boxes can be included in the "strongly heterogeneous assortment" type. Of the total 582 boxes there are 322 different boxes. The assortment of boxes changes depending of the day in analysis.
- Assortment of bins - The assortment of bins is included in the "strongly heterogeneous assortment of large objects" (1064 different bins of a total of 1068)

According to the above presented criteria, the problem is comparable to refined problem type of the 3-dimensional

residual bin packing (3D RBPP), which is described by a strongly heterogeneous set of small items that have to be assigned to a strongly heterogeneous assortment of large objects in order to minimize a function related to the bins.

This problem is different from all C&P problems found in the literature; the guillotine cut constraint, the assortment of boxes' characteristics varies significantly from day to day and the assortment of bins has almost the double of items of the assortment of boxes.

III. CONSTRUCTIVE HEURISTIC

The 3D heuristic algorithm (3DHA) developed by Silveira [1], which is used as the basis for the heuristic and meta-heuristic approaches presented in this work, was inspired by the first-fit decreasing (FFD) type algorithms developed by Crainic [12]. The heuristic was designed for the same problem, with a simpler scrap condition and no chips generation.

This base heuristic uses the Extreme Points (EPs) concept for the definition of packing points, box sorting and EP sorting modules, a packing and direction algorithm and two cutting decision algorithms. EPs provide the means to exploit the free space defined inside a packing by the shapes of the items already in the container [12]. The open guillotine cut (OGC) algorithm is used after the placement of each box in order to find the cutting order which maximizes each extreme point's dimensions and, at the end of the day, a closed guillotine cut (CGC) algorithm is used to obtain the final cutting configurations. In algorithm 1, a simplification of the used heuristic is presented.

Algorithm 1 Simplified 3DHA

```

sort boxes
for all boxes do
    sort EPs
    select the first feasible EP
    packing direction and orientation algorithm
    update EPs
    OGC algorithm
end for
CGC algorithm

```

For a more detailed explanation of the heuristic see [1].

IV. ACO ALGORITHM

The ACO algorithm proposed by Silveira [1], inspired by the work of Levine and Ducatelle [8] for the 1-dimensional bin packing and cutting stock problems, was tested for this extension of the steel cutting problem. This algorithm searches for the best order of boxes and makes use of the constructive heuristic in order to generate solutions.

The used solution construction graph is presented in figure 3. Starting at node 0, where the list of bins is computed, the ant chooses the next node to visit; after a node is visited and the box type packed, the node is made unavailable. When all

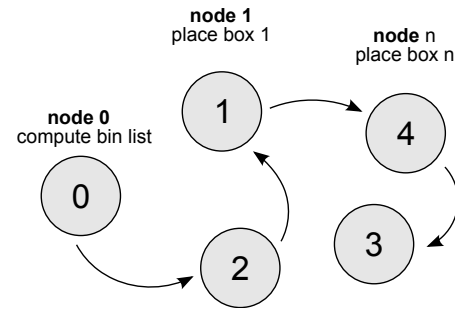


Fig. 3. ACO construction graph with 4 types of boxes

nodes are unavailable the solution is complete, all box types are packed.

This algorithm was tested in order to compare its performance with the proposed approaches. It manipulates the box selection process in contrast to the tabu search and BFD approaches that control the packing point selection.

V. TABU SEARCH ALGORITHM

The proposed tabu search algorithm makes use of the constructive heuristic and was designed to explore multiple packing point options for the different boxes (orders) in order to achieve better results. This algorithm works by changing the packing point selection of the constructive heuristic: instead of choosing the first available extreme-point (EP) the tabu search algorithm will search, for each box, the best EP to use in order to optimize the complete solution.

The algorithm starts by generating the initial solution and its element range. The element range is the method used for the algorithm to bound the solutions generated by the neighborhood generation algorithm to feasible solutions. Having the initial solution, the algorithm generates a large neighborhood and reduces it to a fixed size without having solutions pertaining to the tabu list. The objective function value of the neighborhood solutions is then obtained and the one with best (lower) fitness is chosen as the next iteration's solution and, if its value is higher than the best solution's fitness, the best solution is changed to be equal to the iteration's solution. The cycle finishes by updating the tabu list and starts again generating the next neighborhood.

This cycle is stopped when a fixed number of iterations without improvement of the best solution is achieved. A option for the algorithm to go back to the best solution and running the cycle for another fixed number of iterations after this was included as a way for the algorithm to further explore the best solution neighborhood.

A. Solution Encoding

The used encoding for the tabu search algorithm is used to guide the EP selection process of the constructive heuristic; it represents, for each box (box type or order) the index of the EP, in a sorted list of possible packing points, to be used. This encoding was chosen with the RSCP in mind due to the low number of box types with quantity greater than one and

Algorithm 2 TS Algorithm

N_b : Number of Boxes
 s, s^* : current and best solution
 $f(s)$: scrap generated by solution s
 n, n_b : stopping condition
 m, m_b : Neighborhood size multiplier
 T : Tabu List
 ER : Element range (Range for each solution element)
 $GoBack$: Option to make the algorithm go back to s^*

initialize

- 1) generate initial solution s and obtain ER
- 2) $s^* = s$
- 3) $T = \emptyset, T \leftarrow s$

repeat

generate s neighborhood: $LN(s)$
 $N(s) \leftarrow LN(s)[1 : N_b * m]$
compute $f(s') \quad \forall s' \in N(s)$
 $s = s', f(s') = \min(f(s') \quad \forall s' \in N(s))$
update ER
 $T \leftarrow s$
if $f(s) < f(s^*)$ **then**
 $s^* = s$

end if

until $f(s^*)$ stays unchanged for n iterations

if $GoBack$ **then**

$s = s^*, n = n_b, m = m_b, GoBack = 0$
go to the start of **repeat** cycle

else

stop

end if

easy grouping of boxes. This approach was inspired by the work of Bortfeldt et al. [14], in which a tabu search for the CLP that uses encoded solutions related to the used packing points in an ordered list was proposed.

For a daily instance with N_b number of boxes, an encoded solution s is defined by a vector of N_b integers, encoded solution elements s_i , equal or greater than one. The solution obtained by the constructive heuristic alone, which is used as the initial tabu search solution is then equal to a vector of ones with size N_b .

$$s = (s_1, s_2, \dots, s_{N_b})$$

$$s_{initial} = (1, 1, \dots, 1)$$

Two problems arise with the use of this solution representation; the first one is knowing the limit of each one of the elements of an encoded solution and the second, which almost does not influence the performance due to the low number of cases, is how the algorithm deals with cases where, e.g. the first 4 boxes of a unique box type are packed in the EP given by the encoded box solution eight and, after updating the EPs for the rest of the boxes

of this box type, only seven possible EPs are listed. For the first problem, element range was defined for each one of a solution's elements; the element range of a box type is defined, as the constructive heuristic generates a solution, as the maximum length of the possible EP list. The second problem arises from the former definition, due to the fact that the maximum length is used instead of, e.g., the minimum. This was solved by making the constructive heuristic use the first EP of the list when the encoded solution element is greater than the EP list length.

B. Tabu List

The tabu list is used to escape going back to already selected solutions. The used tabu list is a simple list of the full solutions selected at each iteration. This list is defined with no fixed size due to the fact that the algorithm was designed to run for a low number of iterations.

C. Neighborhood Structure

The neighborhood generation and ordering algorithm generates all the possible solutions with exactly one different element from the parent solution and is bounded by the element range vector ER . The neighborhood is then limited to the $N_b * m_n$ first elements of the total neighborhood. The neighborhood is ordered by the number incremented or subtracted to the initial element, from left to right element increment and increments followed by subtractions.

Example: If $s = (1, 2, 3)$ and $ER = (2, 3, 3)$

$$N = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ 1 & 1 & 3 \\ 1 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Algorithm 3 BFD Algorithm

N_b : Number of Boxes
 s, s^* : current and best solution
 $f(s)$: fitness of solution s
 Max : BFD algorithm solution range

initialize

- 1) generate initial solution s
- 2) $s^* = s$

for $i = 1$ **to** N_b **do**

$N(s^*) = \emptyset$

for $j = 1$ **to** Max **do**

$s' = s^*$

$s'(i) = s'(i) + j$

$N(s^*) = N(s^*) + s'$

end for

compute $f(s') \quad \forall s' \in N(s)$

$s^* = s'$ where $f(s') = \min(f(s') \quad \forall s' \in N(s))$

end for

	Types	Number	Min. Vol	Max. Vol.	\bar{V}	$\delta(V)$	$\frac{\delta}{\bar{V}}$
Boxes	322	582	1.5×10^5	7×10^9	2.3×10^8	6.5×10^8	2.87
Bins	1064	1068	1.7×10^6	1.7×10^{10}	7×10^8	2.4×10^9	3.45

TABLE I
CHARACTERISTICS OF THE BOXES AND BINS

Algorithm	Scrap	Chips Scrap	Total Scrap	Reduction of total scrap [%]	Orders to supp.	Run time (min)
Heuristic	25177.0	13000.0	38177.0	-	130	1.9
ACO*	22906.4	13003.4	35909.8	5.9%	141	16.0
TS	6070.0	13036.0	19106.0	50.0%	85	913.2
BFD	5577.5	12804.0	18381.5	51.9%	90	60.4

* mean of 10 runs ($\sigma(\text{Total Scrap}) = 0.5\%$, $\sigma(\text{Orders to supp.}) = 1.42\%$)

TABLE II
RESULTS FOR EACH ALGORITHM

D. Best-Fit Decreasing Algorithm

A simple best-fit decreasing type algorithm that uses the tabu-search encoding was also developed. It starts by obtaining the initial solution; the algorithm then optimizes each one of the solution's elements at a time, from first to last. Having the initial solutions, it generates a *Max* number of solutions obtained by incrementing the initial solution's first element from 1 to *Max*, the objective function of the solutions is obtained and, the solution with the minimum fitness, if lower than the initial solution, is set as the new initial solution (s^* in the pseudo code); after this the algorithm goes to the next solution element and so on. The workings of the algorithm are presented in algorithm 3.

VI. APPLICATION EXAMPLE

The addressed problem is from an anonymous company from the steel industry with the objective of minimizing the scrap metal generated. The used data corresponds to 3 months of operation and is composed by a list of stock pieces at the end of the operation period, the stock pieces obtained from suppliers added to the stock at a determined day and the daily orders that need to be satisfied.

A. Data

Table I presents the most important characteristics of the problem's data. Volumetric data is presented in mm^3 . \bar{V} represents the average volume. The standard deviation and relative standard deviation (RSD) are also presented. It's possible to verify the heterogeneity of the boxes and bins by observing the RSD' that are greater than 250%. The presented data is proportional to the real data due to confidentiality issues.

B. Results

This section presents the results of the ACO, tabu search and best-fit decreasing algorithm and the comparison to the

performance of the heuristic algorithm. The tabu search algorithm was parametrized with the following parameters: $n = 3$, $n_b = 1$, $m = 5$, $m_b = 30$, *GoBack* = 1. The ACO algorithm uses the following parametrization: 20 ants, 80 iterations or 5 without improvement, $\rho = 0.2$, $\alpha = 1$, $\beta = 2$, $p_{best} = 0.05$, $q = 1$, $p = 2$, iteration best pheromone update. The best-fit decreasing algorithm uses a range of 30. The EP ordering method obtains the lists of increasing d and volume and combines them by setting the odd entries of the used list equal to the d decreasing list and the even entries to the volume decreasing list. The boxes are sorted, for the tabu search algorithm, by increasing d . The aforementioned parameters were chosen in order to obtain the best possible results using the respective algorithms.

In table II, the total weight of scrap residual bins (kg), the chips scrap generated (kg), the total scrap reduction in comparison to the results of 3DHA and the orders to suppliers (standard stock pieces received) used are shown. The BFD algorithm was able to achieve the best performance of the analyzed algorithms being able to reduce the total weight of scrap generated by 52%. The TS algorithm was able to reduce the scrap by 50% but generated greater scrap than the BFD algorithm, showing that, in this problem's case, the correct individual box to bin assignment is more important than the joint analysis of the cutting of multiple orders. The proposed methods outperformed the ACO algorithm that achieved a 7% scrap reduction, showing the importance of testing packings for multiple bins/packing points.

The number of new stock pieces ordered from suppliers that were used was also significantly reduced by the proposed algorithms. The tabu search algorithm reduced the number of orders to suppliers for 35% and the best-fit decreasing 31%, in comparison with the heuristic.

This results show that the chips scrap generated was not significantly affected by the use of different algorithms (reduction of 1.7% by the BFD algorithm and augment of 0.3% by the TS algorithm), meaning that the total cutting are

stayed almost unchanged. The residual bin scrap was able to be greatly reduced by a correct bin choice.

VII. CONCLUSIONS

The approaches proposed in this paper were able to greatly minimize the scrap generated by the addressed steel cutting problem. In comparison to the 3DHA heuristic, which was able to significantly reduce the scrap generated by the company, the developed method reduced the scrap by up to 52%. The addition of the chips generation within the steel cutting process simulation didn't have a high influence on the results for it's value was similar across the different approaches used. The presented results also showed a reduction of orders to suppliers of up to 35% that was obtained from the use of the proposed methods.

Having obtained significantly better results using algorithms that are flexible in terms of the choice of packing point and bin (BFD and TS) in comparison to the ACO algorithm that worked in the box selection space, it can be concluded that the correct box to bin/packing point assignment is of great importance in the scrap minimization.

In the future, more practical aspects experienced in the day-to-day operations of the company should be added to the formulation such as cutting times, handling times, warehouse costs. A multi-objective formulation of the problem should be considered in future work.

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